Ch 12 April 16, 2019

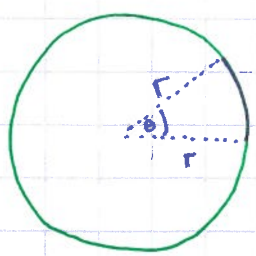
CHAPTER 12: Rotation of a Rigid Body

An object that doesn't change its shape (all parts rotate at the same rate)

initially → Rotation abot a fixed axis

finally → Rolling motion

Rotational Variables	Linear Variables
angular position → θ (rad)	position → x (m)
angular velocity → $\omega = \frac{d\theta}{dt}$ (rad/s)	Velocity → $v = dx/dt$ (m/s)
angular acceleration → $\alpha = \frac{d\omega}{dt}$ (rad/s²)	Acceleration → $a = \frac{dv}{dt}$ (m/s²)



s = arc length

if θ is in radians → $s = r\theta$

* All angles will be measured ccw from the +x-axis

$$\boxed{\theta = \frac{s}{r}}$$

full circle (360°)

$$\theta = \frac{2\pi r}{r} = 2\pi$$

$2\pi \text{ rad} = 360°$

$$\boxed{1 \text{ rad} = \frac{360°}{2\pi} \simeq 57.4°}$$

$2\pi \text{ rad} = 360°$

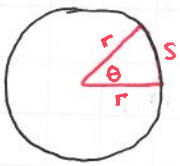
CH 12 April 18, 2019.

RECAP

Angular position → θ → measure ccw from the +x-axis

Angular velocity → $\omega = \frac{d\theta}{dt}$ 360° → $\theta = \frac{2\pi r}{r} = 2\pi$ rad.

angular acceleration → $\alpha = \frac{d\omega}{dt}$ $\boxed{\begin{array}{l} 2\pi \text{ rad} = 360° \\ 1 \text{ rad} \simeq 57.4° \end{array}}$



$s = r\theta$

$\theta = s/r$ θ in rad.

Preview

Every linear quantity/equation we have had so far a corresponding angular position

$x \rightarrow \theta$

$v \rightarrow \omega$

$a \rightarrow \alpha$

$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

↓

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\Sigma \vec{F} = m\vec{a} \rightarrow \Sigma \vec{\tau} = I\alpha$

Torque moment of inertia

$\Sigma \vec{F} = \frac{d\vec{p}}{dt} \rightarrow \Sigma \vec{\tau} = \frac{d\vec{L}}{dt}$ → angular momentum

$\vec{p} = m\vec{v} \rightarrow \vec{L} = I\vec{\omega}$

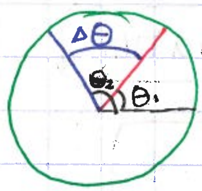
$K = \frac{1}{2}mv^2 \rightarrow K_{tot} = \frac{1}{2}I\omega^2$

Rotational Kinetic Energy

Position x

angular position θ → what angle something is at with respect to the origin (+x-axis)

angular displacement → $\boxed{\Delta\theta = \theta_2 - \theta_1}$ $\Delta\theta > 0$ if CCW $\Delta\theta < 0$ if CW



$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$

$$\boxed{\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}}$$

CCW rotations are +
CW rotations are −

$[\omega] = \frac{rad}{s}$

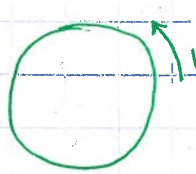
$\frac{1\,rad}{s} = ?\frac{rev}{min}$

$\frac{1\,rad}{s} = \frac{1\,rad}{s}\left(\frac{1\,rev}{2\pi\,rad}\right)\left(\frac{60\,s}{1\,min}\right) \simeq 9.5\,rpm$

$\alpha_{avg} = \frac{\Delta\omega}{\Delta t}$ $[\alpha] = \frac{rad}{s^2}$

$$\boxed{\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}}$$

- α & ω have same sign if speeding up
- α & ω have opposite signs if slowing down



ω slowing down
$\alpha < 0$

Is it possible for different part of a rigid rotating body to have different values of $\Delta\theta$, ω, & α?

NO → All parts of a rigid rotating object have the same $\Delta\theta$, ω, & α.

CH 12

April 18, 2019

- $\vec{w}$ is the slope of θ vs. t
- $\vec{\alpha}$ is the slope of w vs t

→ the area under a curve of w vs. t gives $\Delta\theta$
→ the area under a curve of α vs. t gives Δw

Equations of constant Angular Acceleration

$V_x = V_{0x} + a_x t$ → $w = w_0 + \alpha t$

$x = x_0 + V_{0x} t + \frac{1}{2} a_x t^2$ → $\theta = \theta_0 + w_0 t + \frac{1}{2}\alpha t^2$

$V_x^2 = V_{0x}^2 + 2a_x(x - x_0)$ → $w^2 = w_0^2 + 2\alpha(\theta - \theta_0)$

$x - x_0 = \frac{1}{2}(V_{0x} + V_x)t$ → $\theta - \theta_0 = \frac{1}{2}(w_0 + w)t$

Problem
4.43

part I

$\theta_0 = 0 \text{ rad}$
$\theta = ?$
$w_0 = 0 \text{ rad/s}$
$w = 500 \text{ rev/min} \times \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right) = 52.36 \text{ rad}$
$\alpha = ?$
$t = 1.0 \text{ s}$

$\theta - \overset{=0}{\theta_0} = \frac{1}{2}(\overset{=0}{w_0} + w)t$

$\theta = \frac{1}{2}(wt)$

$= \frac{1}{2}(52.6 \frac{\text{rad}}{\text{s}})(1.0 \text{ s}) \rightarrow \underline{26.2 \text{ rad}}$

part II

$\theta_0 = 0 \text{ rad}$
$\theta = ?$
$w_0 = 52.36 \text{ rad/s}$
$w = 52.36 \text{ rad/s}$
$\alpha = 0 \text{ rad/s}^2$
$t = 3.0 \text{ s}$

CH 12 April 18, 2019

part 2

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$\theta = \omega_0 t$$

$$\theta = (52.36\,rad/s)(3.0\,s) = 157.1\,rad.$$

part 3

$\theta_0 = 0\,rad$

$\theta = ?$

$\omega_0 = 52.36\,rad/s$

$\omega = 0\,rad/s$

$\alpha = ?$

$t = 2.0\,s$

$$\theta - \theta_0 = \tfrac{1}{2}(\omega_0 + \omega)t$$

$$\theta = \tfrac{1}{2}\omega_0 t$$

$$= \tfrac{1}{2}(52.6\,rad/s)(2.0\,s)$$

$$= 52.4\,rad$$

$$\theta = 26.2\,rad + 157.1\,rad + 52.4\,rad = 235.7\,rad \left| \frac{1\,rev}{2\pi\,rad} \right.$$

$$\boxed{\theta = 37.5\,rev}$$

CH 12 + Recap April 18, 2019

part 2

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\theta = \omega_0 t$

$\theta = (52.36 \text{ rad/s})(3.0\text{ s}) = 157.1 \text{ rad}.$

part 3

$\theta_0 = 0 \text{ rad}$

$\theta = ?$

$\omega_0 = 52.36 \text{ rad/s}$

$\omega = 0 \text{ rad/s}$

$\alpha = ?$

$t = 2.0\text{ s}$

$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$

$\theta = \frac{1}{2}\omega_0 t$

$= \frac{1}{2}(52.6 \text{ rad/s})(2.0\text{ s})$

$= 52.4 \text{ rad}$

$\theta = 26.2 \text{ rad} + 157.1 \text{ rad} + 52.4 \text{ rad} = 235.7 \text{ rad} \left| \frac{1 \text{ rev}}{2\pi \text{ rad}} \right.$

$\boxed{\theta = 37.5 \text{ rev}}$

Recap

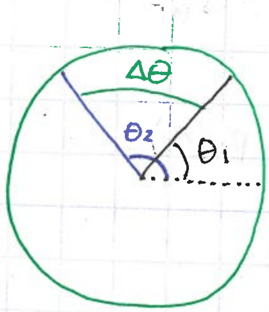
April 22, 2019

Rotational or Angular Quantities:

θ (rad)

ω (rad/s)

α (rad/s²)



$\Delta\theta = \theta_2 - \theta_1$

$\boxed{\omega_{ave} = \frac{\Delta\theta}{\Delta t}}$

$\boxed{\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}}$

$\boxed{\alpha_{ave} = \frac{\Delta\omega}{\Delta t}}$

$\boxed{\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}}$

CH 12 April 18, 2019

- $\vec{\omega}$ is the slope of θ vs. t
- $\vec{\alpha}$ is the slope of ω vs. t

→ the area under a curve of ω vs. t gives $\Delta\theta$
→ the area under a curve of α vs. t gives $\Delta\omega$

$\boxed{\text{Equations of constant Angular Acceleration}}$

$V_x = V_{0x} + a_x t \longrightarrow \omega = \omega_0 + \alpha t$

$x = x_0 + V_{0x} t + 1/2 a_x t^2 \longrightarrow \theta = \theta_0 + \omega_0 t + 1/2 \alpha t^2$

$V_x^2 = V_{0x}^2 + 2a_x(x - x_0) \longrightarrow \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$x - x_0 = 1/2(V_{0x} + V_x)t \longrightarrow \theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$

Problem 4.43

part I

$\theta_0 = 0 \text{ rad}$

$\theta = ?$

$\omega_0 = 0 \text{ rad/s}$

$\omega = 500 \text{ rev/min} \times \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right) = 52.36 \text{ rad}$

$\alpha = ?$

$t = 1.0 s$

$\theta - \overset{=0}{\theta_0} = \frac{1}{2}(\overset{=0}{\omega_0} + \omega)t$

$\theta = \frac{1}{2}(\omega t)$

$= \frac{1}{2}(52.6 \frac{rad}{s})(1.0s) \longrightarrow \underline{26.2 \text{ rad}}$

part II

$\theta_0 = 0 \text{ rad}$ $\omega = 52.36 \text{ rad/s}$

$\theta = ?$ $\alpha = 0 \text{ rad/s}^2$

$\omega_0 = 52.36 \text{ rad/s}$ $t = 3.0 s$

Recap
+
CH. 12

April 22, 2019

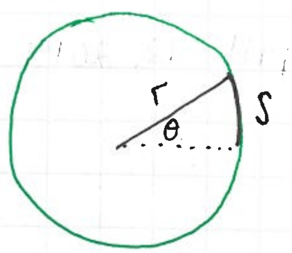
$$\boxed{\begin{aligned} \omega &= \text{slope of } \theta \text{ vs. } t \\ \alpha &= \text{slope of } \omega \text{ vs. } t \\ \Delta\theta &= \text{area under curve of } \omega \text{ vs. } t \\ \Delta\omega &= \text{area under curve of } \alpha \text{ vs } t. \end{aligned}}$$

* $\Delta\theta$, ω, & α are the same for all parts of a rotating, rigid body

$$\boxed{\begin{aligned} \omega &= \omega_0 + \alpha t \\ \theta &= \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2 \\ \omega^2 &= \omega_0^2 + 2\alpha(\theta - \theta_0) \\ \theta - \theta_0 &= \tfrac{1}{2}(\omega + \omega_0)t \end{aligned}}$$

only apply if α is constant.

How are linear & rotational quantities related?



$\theta = \frac{s}{r}$ (θ in radians)

$s = r\theta \rightarrow \frac{ds}{dt} = \frac{d}{dt}(r\theta)$

$\underbrace{\frac{ds}{dt}}_{V_t} = r\underbrace{\frac{d\theta}{dt}}_{\omega}$

V_t (tangential speed)

$m/s = m\,(rad/s)$

$\boxed{V_t = r\omega}$

$dV_t/dt = d/dt\,(r\omega)$

$dV_t/dt = r\frac{d\omega}{dt}$

$a_t = r\alpha$

- $V_t \rightarrow$ tangential speed in m/s
- $r \rightarrow$ distance from the x-axis in m
- $\omega \rightarrow$ angular speed in rad/s

CH. 12 April 22, 2019

$\frac{dV_t}{dt} = r\frac{d\omega}{dt} = \boxed{a_t = r\alpha}$ → acceleration tangent to the path which has to do with a change of speed.

$a_r = \frac{v^2}{r}$ ⟶ acceleration because of a change in direction (towards center of circular path)

$a_r = \frac{v^2}{r} = \frac{(r\omega)^2}{r}$

$\boxed{a_r = r\omega^2}$

total acceleration: $\boxed{a = \sqrt{a_t^2 + a_r^2}}$

1Q: A wheel starts from rest and spins with a constant angular acceleration. As time goes on the acceleration vector for a point on the rim:

D) Increase in magnitude and becomes more nearly radial.

Center of mass (COM) ⟶ The average position of the mass of an object.

⟶ An unconstrained object (not on an axle or pivot) on which there is no net force will rotate about center-of-mass (COM)

$$X_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{M}$$

M → total mass : $m_1 + m_2 + m_3 + \ldots$

$$Y_{cm} = \frac{m_1 Y_1 + m_2 Y_2 + m_3 Y_3 + \cdots}{M}$$

M → total mass : $m_1 + m_2 + \cdots$

↳ TRUE for a set of particles.

CH. 12 April , 2019

For solid objects:

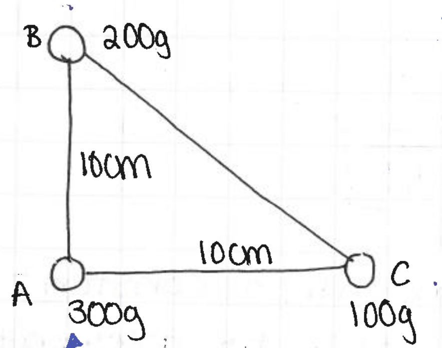
$$\boxed{\begin{aligned} X_{cm} &= \frac{1}{M}\int x\,dm \\ Y_{cm} &= \frac{1}{M}\int y\,dm \end{aligned}}$$

$\rho = \frac{m}{V}$

$m = \rho V$ if ρ = constant

$dm = \rho dV$

PROBLEM 12-6



set A as origin
$A_x = 0$
$A_y = 0$

$$X_{cm} = \frac{m_A X_A + m_B X_B + m_C X_C}{m_A\ m_B + m_C}$$

$$X_{cm} = \frac{(300g)(0cm) + (200g)(0cm) + (100g)(10cm)}{300g + 200g + 100g}$$

$X_{cm} = 1.7cm$

$$Y_{cm} = \frac{m_A Y_A + m_B Y_B + m_C Y_C}{m_A + m_B + m_C}$$

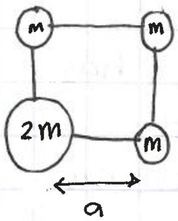
$$Y_{cm} = \frac{(300g)(0cm) + (200g(10cm) + (100g)(0cm)}{300g + 200g + 100g}$$

$Y_{cm} = 3.3cm$

SET origin at C:

$$X_{cm} = \frac{(300g)(-10cm) + (200g)(-10cm) + (100g)(0cm)}{300g + 200g + 100g}$$

$X_{cm} = -8.3cm$



What is the x-coordinate of the center of mass?

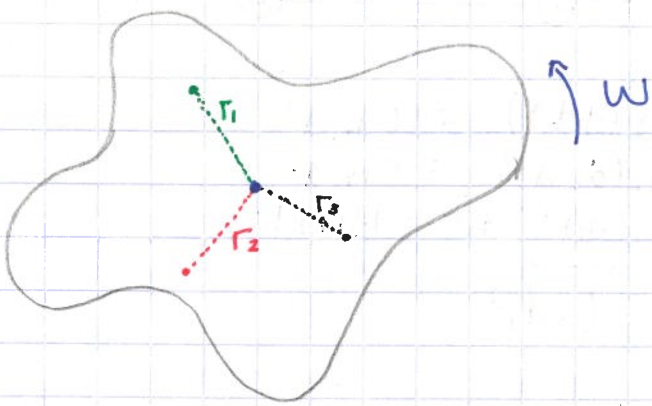
$\frac{2}{5}a$

$$\frac{2m(\cancel{0cm})^{0} + m(\cancel{0cm})^{0} + m(a) + m(a)}{2m+m+m+m} = \frac{2\cancel{m}a}{5\cancel{m}} = \frac{2}{5}a$$

Rotational Energy

→ A rotating object has kinetic energy, even if it's com is not moving translationally

↳ Each particle that makes up the object is moving around the axis



Looking at 3 of a bazillion particles that make up the object.

$$K_{rotation} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2 + \dots$$

$$= \frac{1}{2}m_1(r_1w)^2 + \frac{1}{2}m_2(r_2w)^2 + \frac{1}{2}m_3(r_3w)^2 + \dots$$

$$= \frac{1}{2}m_1r_1^2w^2 + \frac{1}{2}m_2r_2^2w^2 + \frac{1}{2}m_3r_3^2w^2 + \dots$$

$$= \frac{1}{2}(m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \dots)w^2$$

$$= \frac{1}{2}\left(\sum_{i=1}^{n} m_ir_i^2\right)w^2$$

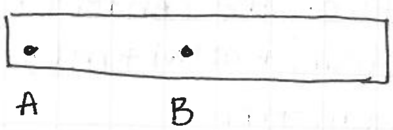
→ Moment of Inertia (I)

→ Rotational equivalent of mass.

$$\boxed{I = \sum_{i=1}^{n} m_ir_i^2}$$

→ You must give I relative to an axis of rotation.

April 22, 2019



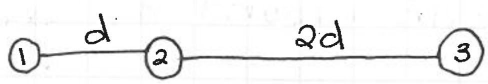
$I_A > I_B$

* The more mass an object has farther from the axis of rotation, the more rotational inertia it has (greater moment of inertia)

IQ:

Then rank the spheres according to the rotational inertia about them, greatest first.

c) 3, 1, 2



$1 \rightarrow I = 0 + md^2 + m(9d^2) = 10md^2$

$2 \rightarrow I = md^2 + 0 + m(4d^2) = 5md^2$

$3 \rightarrow I = m(9d^2) + m(4d^2) + 0 = 13md^2$

Calculating I

$I = \sum_{i=1}^{n} m_i r_i^2$ is only useful for a collection of particles

↳ a continious object, the sum becomes an integral $I = \int r^2 dm$

moment of inertia you should know:

Cylinder or disk about a center

CH. 12

RECAP

April 23, 2019

→ Relative Linear & Angular Quantities:

change in speed $\boxed{V_t = r\omega}$ $\boxed{V_r = 0}$ $\boxed{a_t = r\alpha}$ $\boxed{a_r = v^2/r = r\omega^2}$ change in direction

total acceleration: $\boxed{a = \sqrt{a_t^2 + a_r^2}}$

→ Center of mass (C.O.M): average position of the mass of an object

$$\boxed{\begin{aligned} X_{cm} &= \frac{1}{M}\sum_{i=1}^{n} m_i x_i = \frac{m_1x_1 + m_2x_2 + \dots}{m_1 + m_2 + \dots} \\ Y_{cm} &= \frac{1}{M}\sum_{i=1}^{n} m_i y_i = \frac{m_1y_1 + m_2y_2 + \dots}{m_1 + m_2 + \dots} \end{aligned}}$$

Solid objects: $X_{cm} = \frac{1}{M}\int x\,dm$

$Y_{cm} = \frac{1}{M}\int y\,dm$

Rotational Energy

$\boxed{K_{rot} = \frac{1}{2}\left(\sum_{i=1}^{n} m_i r_i^2\right)\omega^2}$ → Moment of Inertia I (rotational inertia)

↳ Rotational analog of mass

$\boxed{I = \sum_{i=1}^{n} m_i r_i^2}$ → I depends on how the mass is distributed relative to the axes of rotation

$\boxed{[I] = kg\,m^2}$

* Must be given relative to an axis

→ For a continious object, the sum becomes an integral: $\boxed{I = \int r^2 dm}$

$\boxed{K_{rot} = \frac{1}{2} I \omega^2}$

IQ: A uniform solid cylinder made of lead has the same mass and the same length as a uniform solid cylinder made of wood. The rotational inertia of the lead cylinder compared to the wooden one is: less b/c $\rho = \frac{m}{V}$ ← same

lead has greater density

Moments of Inertia from table 12-2:

- Cylinder or disk about the center $I = \frac{1}{2} MR^2$
  (will use for pulleys)
- Solid sphere about diameter $I = \frac{2}{5} MR^2$
- Spherical shell about diameter $I = \frac{2}{3} MR^2$
- Cylindrical hoop about center $I = MR^2$
- Thin rod about center $I = \frac{1}{12} ML^2$
- This rod about end $I = \frac{1}{3} ML^2$
- Plane or slab about the center $I = \frac{1}{2} Ma^2$
- Plane or slab about edge: $I = \frac{1}{3} Ma^2$

CH: 12 April 23, 2019

Problem 12.10

$M_{earth} = 5.98 \times 10^{24}$ kg
$R_{earth} = 6.37 \times 10^{6}$ m
$T_{earth} = 24$ Hrs
$= 86,400$s

$K_{rot} = \frac{1}{2} I \omega^2$

$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{rad}}{86,400\text{s}} = 7.27 \times 10^{-5} \frac{\text{rad}}{\text{s}}$

$$K_{tot} = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2$$
$$= \frac{1}{5}MR^2\omega^2$$
$$= \frac{1}{5}(5.98 \times 10^{24}\text{kg})(6.37 \times 10^{6}\text{m})^2\left(7.27 \times 10^{-5}\frac{\text{rad}}{\text{s}}\right)^2$$
$$= 2.57 \times 10^{29} \text{J}$$

How do we affect I of the earth?

$I = \sum mr^2 \rightarrow$ For a collection of point masses

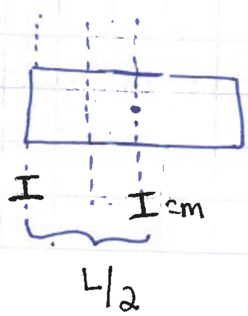
* Moments of inertia add together like masses

$$I_{earth + people} = I_{earth} + \sum I_{people}$$
$$+ M_{person}(R_{person})^2(\#people)$$

$$= I_{earth} + (60\text{kg})(6.37 \times 10^{6}\text{m})^2(7 \times 10^{9})$$
$$= I_{earth} + 1.70 \times 10^{25} \text{kgm}^2$$

Parallel-Axis Theorem

↳ Useful for calculating I about an axis parallel to an axis passing through the center of mass (I_{cm})



$$I = I_{cm} + Md^2$$

↳ d - distance from a parallel axis through the center of mass

$$I = \tfrac{1}{12} ML^2 + M(L/2)^2$$
$$= \tfrac{1}{12} ML^2 + \tfrac{1}{4} ML^2$$
$$= \boxed{\tfrac{1}{3} ML^2}$$ → Thin rod with axis at one end

Rolling Motion

↳ The motion of a rolling object is a combination of translational motion (of the com) and rotational motion (about an axis passing through the com)

* If something is rolling smoothly (without slipping), then the C.O.M. moves forward a distance of $s = r\theta$

Speed of C.O.M → $\frac{ds}{dt} = r\frac{d\theta}{dt} = \boxed{V_{cm} = R\omega}$

$\boxed{V_{cm} = R\omega}$ → speed C.O.M. moves forward
↳ Also the speed of the point at the edge of the wheel that is just rotating

Kinetic Energy of Rolling

↳ The Kinetic Energy of a rolling object has two parts:

1) Translational kinetic Energy of the C.O.M
$K_{cm} = \frac{1}{2} mV_{cm}^2$

2) Rotational kinetic energy about an axis through the C.O.M
$K_{rot} = \frac{1}{2} I_{cm} \omega^2$

CH 12. April 23, 2018

$$\boxed{K_{rolling} = \frac{1}{2} m V_{cm}^2 + \frac{1}{2} I_{cm} \omega^2}$$

- $V_{cm} = R\omega$

  R - radius of wheel

  ω - angular speed

Solid Sphere rolling

$$K = \frac{1}{2} m V_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$
$$= \frac{1}{2} m V_{cm}^2 + \frac{1}{2}\left(\frac{2}{5} MR^2\right)\left(\frac{V_{cm}}{R}\right)^2$$
$$= \frac{1}{2} m V_{cm}^2 + \frac{1}{5} m V_{cm}^2$$
$$= \frac{7}{10} m V_{cm}^2$$

Problem 12-A

System → Spring, earth & ball

External forces → Normal force

$W_{ext} = 0$

$$\overset{=0}{(K_{cm})_f} + \overset{=0}{(K_{rot})_f} + (U_g)_f + \overset{=0}{(U_{sp})_f} + \overset{=0}{\Delta E_{th}} = \overset{=0}{(K_{cm})_i} + \overset{=0}{(K_{rot})_i} + \overset{=0}{(U_g)_i} + (U_{sp})_i + \underset{=0}{W_{ext}}$$

$$mgy_f = \frac{1}{2} k (\Delta x)^2$$

$$y_f = \frac{\frac{1}{2} k (\Delta x)^2}{mg} = 0.92\,m \qquad d = \frac{(0.92\,m)}{\sin 30°} = 1.84\,m$$

(b) $\frac{1}{2} m V_{cm}^2 + \frac{1}{2} I \omega^2 + mgy_f = \frac{1}{2} k (\Delta x_i)^2$

$$\frac{1}{2} m V_{cm}^2 + \frac{1}{2}\left(\frac{2}{5} MR^2\right)\left(\frac{V_{cm}}{R}\right)^2 + mgy_f = \frac{1}{2} k (\Delta x_i)^2$$

$$\frac{7}{10} m V_{cm}^2 + mgy_f + \frac{1}{2} k (\Delta x_i)^2$$

$$y_f = (1.2 cm) \sin 30° = 0.60\,m$$

$$V_{cm} = 2.1\,m/s$$

Recap + CH12 April 25, 2019

rotational kinetic Energy $\boxed{K_{rot} : \frac{1}{2} I \omega^2}$

Note: → Angular momentum Not in 2nd ExAM

moment of Inertia: $\boxed{I = \Sigma m r^2 = \int r^2 dm}$

* Table 12-2 for moments of inertia

Parallel-Axis Theorem: $\boxed{I = I_{cm} + Md^2}$

Rolling Motion → combination of pure translational motion & pure rotational motion

$\boxed{V_{cm} = R\omega}$ speed of C.O.M for smooth rolling
↳ Also speed of a point on the edge of the rolling object.

Kinetic energy of rolling:

$$\boxed{K_{rolling} = K_{cm} + K_{rot} = \frac{1}{2} m V_{cm}^2 + \frac{1}{2} I \omega^2}$$

Problem 12-B

$$(K_{cm})_f + (K_{rot})_f + \underbrace{(U_g)_f}_{=0} + \underbrace{(U_{sp})_f}_{=0} + \underbrace{\Delta E_{th}}_{=0} = \underbrace{(K_{cm})_i}_{=0} + \underbrace{(K_{rot})_i}_{=0} + (U_g)_i + \underbrace{(U_{sp})_i}_{=0} + \underbrace{W_{ext}}_{=0}$$

System:
ball + Surface + Earth $W_{ext} = 0$
y_0 = at end of ramp

Problem 12-B continued...

$$\frac{1}{2}mV_{cm}^2 + \frac{1}{2}I\omega^2 = mgy_i \longrightarrow \frac{1}{2}mV_{cm}^2 + \frac{1}{5}mV_{cm}^2 = mgy_i$$

$$\frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(V_{cm}/R\right)^2 = \frac{1}{5}mV_{cm}^2$$

$$\frac{7}{10}MV_{cm}^2 = mgy_i$$

$$V_{cm} = \sqrt{\frac{10}{7}gy_i}$$

$$V_{cm} = \sqrt{\frac{10}{7}(9.80\,\tfrac{m}{s^2})(4.0\,m)}$$

$$V_{cm} = 7.48\,m/s$$

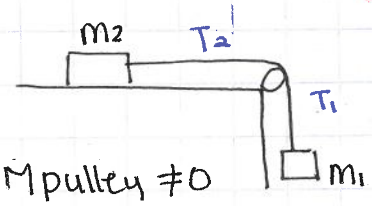
→ Projectile motion.

Torque ($\vec{\tau}$) → "Turning force"

Rotational analog of force

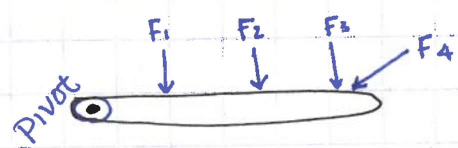
$$\Sigma\vec{F} = m\vec{a} \longrightarrow \boxed{\Sigma\vec{\tau} = I\vec{\alpha}}$$

* If a pulley is NOT massless, then the tension on each side of the pulley will NOT be the same.



if $M_{pulley} \neq 0$

$T_2 \neq T_1$



what affects τ?

- force $\vec{F}$
- distance of force from axis of rotation $\vec{r}$
- angle between $\vec{F}$ and $\vec{r}$

$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$ cross product $\vec{r} \times \vec{F}$

$$\boxed{\tau = rF\sin\phi}$$ magnitude of τ

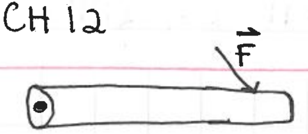
F → Force in N

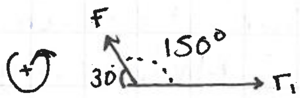
r → distance in m from axis of rotation to $\vec{F}$

ϕ → angle between $\vec{r}$ and $\vec{F}$

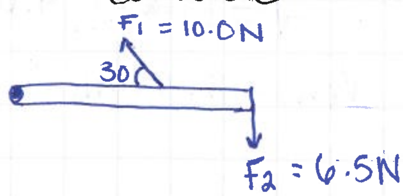
↳ points from axis of rotation to $\vec{F}$

CH 12 April 25, 2019





A torque that produces a ccw rotation is considered +



$r_1 = 75cm$

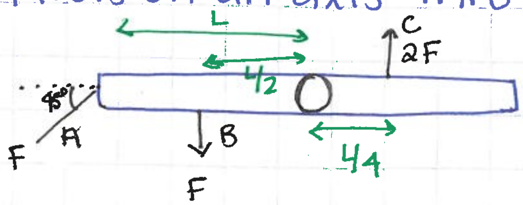
$r_2 = 100cm$

what is $\Sigma \vec{T}$?

$$\Sigma T = r_1 F_1 \sin\theta_1 - r_2 F_2 \sin\theta_2$$
$$= 0.75m(10.0N)\sin 150° - (1.00m)(6.5N)(\sin 90°)$$
$$= \underline{-2.75Nm}$$

CH 12 April, 25, 2019

1Q. Three forces labeled A, B, C, are applied to a rod which pivots on an axis thru its center. [$\cos 45° = \sin 45° = 1/\sqrt{2} = 1/1.41$



$A \rightarrow LF \sin 45° = 0.717 LF$

$B \rightarrow (\frac{L}{2}) F = 0.5 LF$

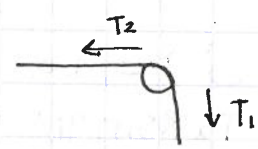
$C \rightarrow (\frac{L}{4})(2F) = \frac{LF}{2} = 0.5 LF$

Which force causes the largest magnitude torque?

A

Newton's 2ND Law of Rotations

↳ State without proof

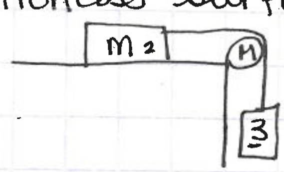


$$\boxed{\Sigma \tau = I\alpha}$$

*key point: If mass of pulley ≠ 0, then tensions are not equal / same linear acceleration of the rope & angular acceleration of the pulley are related by

$$\boxed{a_t = R\alpha}$$

Frictionless surface

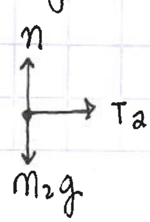


massless pulley:

$a = \frac{m_1 g}{m_1 + m_2}$

- What are T_1 and T_2?
- What is magnitude of accelerations of the block?

block 2:

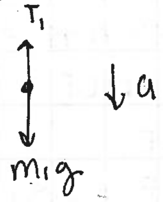


$\Sigma F_x = ma_x$

$\boxed{T_2 = m_2 a}$

CH 12 April 25, 2019

block 1:

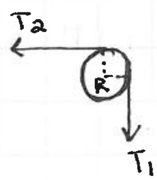


$\Sigma F_y = ma_y$

$m_1 g - T_1 = m_1 a$

$\boxed{T_1 = m_1 g - m_1 a}$

Pulley:



* If using the equation $a_t = r\alpha$, then a +(pos) linear acceleration must result in a positive angular acceleration.

$\Sigma \tau = I\alpha$

$T_2 R \sin 90° - T_1 R \sin 90° = I\alpha$

$(T_1 - T_2)R = \left(\frac{1}{2}MR^2\right)\left(\frac{-a_t}{R}\right)$

FINAL EXAM

CH 12 : Angular momentum

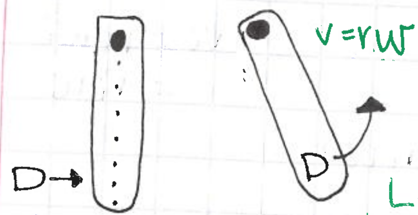
May 2, 2019

(Linear) momentum $\boxed{\vec{p} = m\vec{v}}$

$$\boxed{\text{If } \quad \Sigma \vec{F}_{ext} = 0 \qquad \frac{d\vec{p}}{dt} = 0 \qquad \vec{P}_f = \vec{P}_i}$$

* very useful for collisions b/c mechanical energy is not conserved.

↳ Conservation of angular momentum



angular momentum: $\boxed{L = I\omega}$

↳ Rigid body rotating about a fixed axis.

$L_f = (I_{total})\omega_f$

↳ This moving particle must have angular momentum even though it is Not "rotating"

angular momentum for a moving particle:

$$\boxed{\begin{aligned} \vec{L} &= \vec{r} \times \vec{p} \\ &= \vec{r} \times m\vec{v} \\ &= m(\vec{r} \times \vec{v}) \end{aligned}}$$

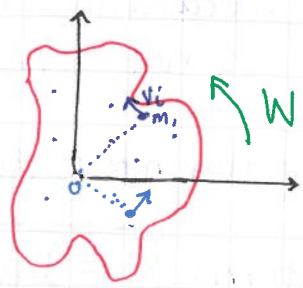
* $\vec{r}$ → Position vector that points from axis of rotation (or origin) to particle

* $\vec{L}$ → must always be given with respect to an origin or axis of rotation.

FINAL EXAM

CH. 12 May 2, 2019

Angular momentum of a rotating rigid body:



$L = \sum_i L_i$

$= \sum_i (r_i m_i v_i \sin\theta_i)$ $\theta = 90°$

$= \sum_i r_i m_i v_i$ $v_i = r_i \omega_i$
$= r_i \omega$

$= \sum r_i m_i (r_i \omega)$

$L = (\sum m_i r_i^2)\omega = I\omega$

$\boxed{\sum \vec{\tau} = I\vec{\alpha}}$ Newton's 2nd law for Rotations

$\sum \vec{F} = \frac{d\vec{p}}{dt} \longrightarrow \boxed{\sum \tau = \frac{d\vec{L}}{dt}}$

Conservation of Angular momentum

$\boxed{\text{if } \sum \vec{\tau}_{ext} = 0 \text{ then } \frac{dL}{dt} \text{ so } \vec{L}_f = \vec{L}_i}$

Problem 12-D

1) Use cons of Energy to get v of block
2) Use cons. of L to get ω
3) use cons. of Energy to get height (& then angle)

1) $mgy_i = \frac{1}{2} m v_f^2$

$v_f = \sqrt{2gy_i} = 1.98 \, m/s^2$

2) System → block + rod

$\vec{L} = \vec{r} \times m\vec{v}$
$= rmv\sin\theta$

$\Sigma T_{ext} = 0 \qquad L_i = L_f$

$L_{block} + L_{rod} = (I_{rod+block})\,\omega_f$
$\downarrow \qquad\qquad = 0$

$rmv\sin\theta = (I_{rod+block})\,\omega_f$

$Lmv\sin 90° = (I_{rod+block})\,\omega_f$

$Lmv\sin 90° = (\frac{1}{3}ML^2 + mr^2)\,\omega_f$

$Lmv\sin 90 = (\frac{1}{3}mL^2 + mL^2)\,\omega^2$

$Lmv = (\frac{4}{3}mL^2)\,\omega^2$

$\underline{\omega_f = 2.97 \text{ rad/s}}$

3) $\frac{1}{2} I_{rod+block}\,\omega^2 = Mg\underset{\nearrow rod}{Y_{cm}} + mg\underset{\nearrow block}{Y_{cm}}$